A Study of Shear Alfvén Waves in Magnetic Stars: the Spherical Shell Model

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Abstract. We carry out an investigation of the propagation of axisymmetric poloidal shear Alfvén waves in a spherical shell of resistive plasma with a background dipolar magnetic field. Using a numerical iterative method to solve this eigenvalue problem, we show that the energy of such waves is trapped in the magnetic polar neighbourhood. This observation may specially be of interest for the asteroseismology of magnetic stars. Some other new features have been discovered, such as thin internal shear layers due to resonant magnetic field-lines.

Keywords: MHD: oscillations, stars: roAp

1. Introduction

In recent years, the problem of the asteroseismology of magnetic stars has mainly been studied from the point of view of acoustic oscillations pertubated by a permanent magnetic field (Bigot et al., 2000). An other approach to the problem deals with the indirect effect of the magnetic field on the driving of the oscillations (Balmforth et al., 2001). roAp's pulsational properties were until recently described by the so-called oblique pulsator model (Kurtz, 1990), but it is still an open question whether the magnetic axis is the same as the pulsation axis in these stars (Bigot and Dziembowski, 2002).

Motivated by the conclusion that regular perturbation methods are not valid in stellar atmospheres where low values of the β-parameter of the plasma are found, and that the model of Balmforth et al. does not explicitly take into account the Lorentz force, we consider a simple situation involving a spherical layer of incompressible resistive plasma submitted to a strong dipolar magnetic field. Of course, this outrageously simplified model does not allow to determine the eigenfrequencies of roAp stars, as the real problem is far more complicated. The main purpose of this idealization is to clarify the role played by the dipole field in the spatial structure of Alfvén modes.

2. Definition of the eigenvalue problem

2.1. GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

The oscillations in shear Alfvén modes of an incompressible plasma layer obey the following set of non-dimensional equations

$$\begin{cases} \lambda\,\vec{\nabla}\times\vec{v} &= \vec{\nabla}\times((\vec{\nabla}\times\vec{b})\times\vec{B}) + \mathrm{E}\,\vec{\nabla}\times\Delta\vec{v} \\ \lambda\,\vec{b} &= \vec{\nabla}\times(\vec{v}\times\vec{B}) + \mathrm{E_m}\Delta\vec{b} \\ \vec{\nabla}\cdot\vec{v} &= 0 \\ \vec{\nabla}\cdot\vec{b} &= 0 \end{cases} \tag{1}$$

where $\mathrm{E} = \nu/(RV_\mathrm{A}) \sim 10^{-13}$ characterizes the diffusivity effects of radiative viscosity, $\mathrm{E_m} = \nu_\mathrm{m}/(RV_\mathrm{A}) \sim 10^{-8}$ is the analogous for ohmic dissipation and V_A is the Alfvén velocity. Both numbers measure the non-adiabaticity of the layer. In the following, $\vec{x} = (\vec{v},\vec{b})$ is an eigenvector composed by the velocity and magnetic perturbations, and $\lambda = i\omega + \tau$ is the associated eigenvalue.

In addition to the previous equations we use the following boundary conditions. On the upper boundary we impose that tangential stresses vanish and require the continuity of the magnetic field with a potential external field. On the inner boundary, we also use stress-free conditions and freeze the dipolar magnetic field by imposing an infinitely conducting core. We are thus faced with a generalized eigenvalue problem of the form $\mathcal{A}\vec{x} = \lambda\mathcal{B}\vec{x}$, where the $\mathcal{A}$ and $\mathcal{B}$ are differential operators.

2.2. PROJECTION AND SYMMETRIES

We project the radial part of the problem on a Gauss-Lobatto grid with a lot of mesh points near the boundaries so that boundary layers can be resolved more efficiently.

The angular part of the problem is projected on the spherical harmonics base. The effect of the magnetic field is then to couple the l and $l+1$ components together, so that it is not possible to characterize the mode by two quantum numbers (n,l) as is often the case in asteroseismology. In the axisymmetrical case ($m = 0$), which will be the only one discussed here, the poloidal components are decoupled from the toroidal ones, which reduces significantly the size of the numerical problem. Another property of the axisymmetrical case is that a global parity of the mode with respect to the star's equator can be defined, and that this parity is opposite for the velocity and magnetic perturbations.

3. Axisymmetric poloidal shear Alfvén eigenmodes

3.1. Eigenvalue spectrum and associated eigenfunctions

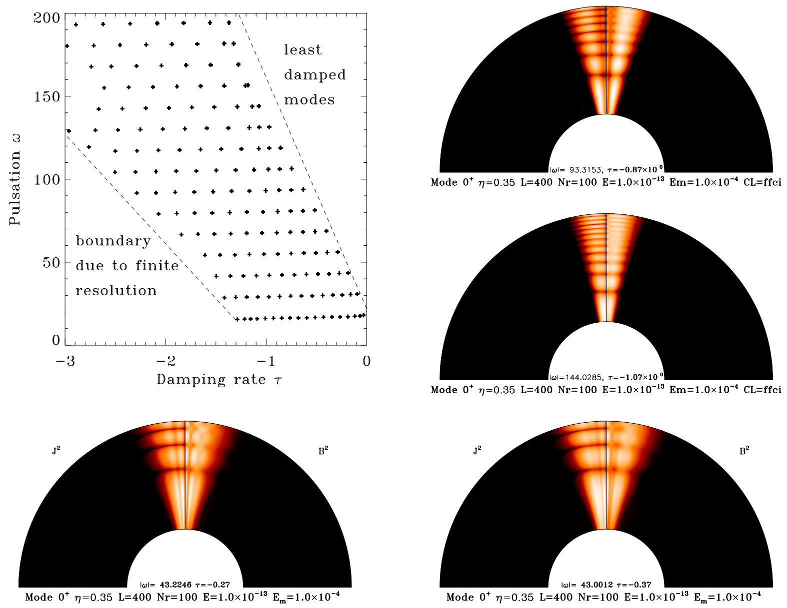

Figure 1. Top left: eigenvalue spectrum of axisymmetric poloidal eigenmodes in the complex plane. The other pictures represent a meridional cut of the distribution of magnetic energy (right quadrants) and dissipation (left quadrants) in the shell for various modes with different periods (or radial wave numbers) and horizontal wave numbers.

The most obvious feature of these modes is their focalization near the magnetic pole. Actually, as can be seen on Fig. 1, their geometry depends on two quantified wave numbers which can roughly be identified as a radial k_r and perpendicular k_θ waves numbers. The top left plot of Fig. 1 shows that ω depends mainly on k_r because near the pole k_r is nearly parallel to the field lines (this is expected from the plane Alfvén waves dispersion relation). Also, these Alfvén waves are dispersive because the Alfvén speed scales as $1/r^3$ on the polar axis for a dipole field.

3.2. Adiabatic versus non-adiabatic features

We explored the evolution of the modes when the various diffusivities where decreased. The polar structure of the mode persists in the adiabatic limit and can be described by low-degree harmonics. On the other hand, some internal singularities appear in this limit because of the presence of shears on resonant field lines, which are an important obstacle for the numerical investigation. In the thin-shell limit internal layers can be observed very well by keeping only the high-degree components of the spherical harmonics decomposition.

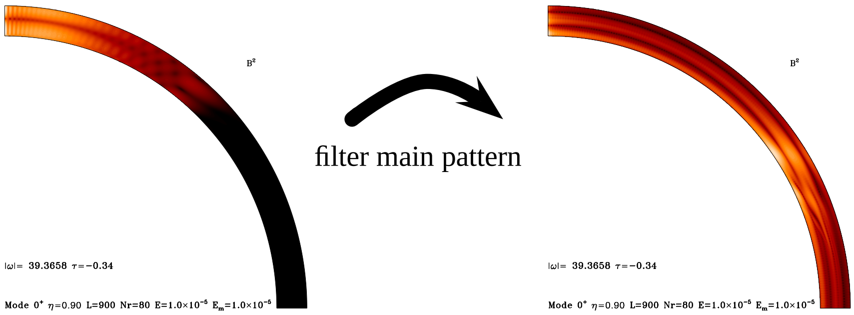

Figure 2. Distribution of magnetic energy in a meridional plane in the thin shell limit. Only the contribution of $l > 300$ harmonics is used in the right picture to underline the numerical difficulties raised by the existence of resonant magnetic field lines (2, 3, etc. nodes instead of one single node near the pole).

4. Conclusions

In this paper we presented some properties of shear Alfvén waves in spherical geometry, when a dipolar magnetic field was present. Using quite a crude model, it is possible to shed some light on interesting features like focalization near the poles and internal layers. The next steps in this study will be to consider toroidal and non-axisymmetric oscillations, so as the coupling between acoustic and magnetic waves, to obtain a more realistic model of roAp star.

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