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SHEAR ALFVÉN MODES IN A MAGNETIZED SPHERICAL SHELL

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Abstract. An investigation of shear Alfvén waves inside a spherical shell is carried out, in which the background magnetic field is dipolar and resistive effects are taken into account. Numerical results indicate two basic behaviours for both the axisymmetric and non-axisymmetric cases. Poloidal modes appear to remain regular, except for internal shear layers, when kinetic and magnetic diffusivities become arbitrarily small, whereas toroidal modes become singular. The corresponding eigenvalues also exhibit different behaviours in the two cases. Analytical results are provided for the axisymmetric toroidal case.
Keywords: MHD – stars: oscillations – stars: magnetic fields

1 Introduction

Numerous astrophysical systems exhibit a pulsating behaviour which may be significantly affected by the presence of a strong magnetic field. roAp stars are probably the most striking examples of such systems and have been the focus of intense work in recent years. A number of complementary models (see Kurtz (1990), Dziembowski & Goode (1996), Balmforth et al. (2001)) have been put forth to explain the dynamics of these stars, but have had to resort to an approximate treatment of the effects of the magnetic field. In this work (see Rincon & Rieutord (2003), Reese et al. (2004)), we aim, instead, at making a more detailed treatment of these effects. This is done at the expense of working with an extremely simplified star.

2 The model

The "star" we work with consists of a spherical shell of incompressible magnetized fluid bathed in a permanent dipolar magnetic field. We then search for oscillation modes in the form of perturbations $\vec{v}$ and $\vec{b}$ to the velocity and magnetic fields

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(resp.) with a temporal dependence $\exp(\lambda t)$. These perturbations are governed by the following set of non-dimensional linearised MHD equations:

$$
\begin{array}{lcl}
\vec{\nabla}\cdot\vec{v} & = & \vec{\nabla}\cdot\vec{b} \quad = \quad 0, \\
\lambda\vec{\nabla}\times\vec{v} & = & \vec{\nabla}\times\left((\vec{\nabla}\times\vec{b})\times\vec{B}\right) + E\vec{\nabla}\times\Delta\vec{v}, \\
\lambda\vec{b} & = & \vec{\nabla}\times\left(\vec{v}\times\vec{B}\right) + E_{\mathrm{m}}\Delta\vec{b}.
\end{array}
\tag{2.1}
$$

where $\vec{B}$ is the permanent dipolar magnetic field, λ is an eigenvalue, and E, E_{m} represent the kinetic and magnetic diffusivities. These typically take on the values $E_{\mathrm{m}} = 10^{-8}$ and $E = 10^{-13}$ (see Rincon & Rieutord (2003)). The perturbations $\vec{v}$ and $\vec{b}$ are composed of an infinite number of spherical harmonics with different degrees ℓ but the same azimuthal order m.

3 Results

Our numerical simulations reveal two types of modes: the "poloidal" type and the "toroidal" one. In the poloidal case, modes seem to be composed of a regular adiabatic part centered on the magnetic poles, and of internal shear layers. These internal shear layers correspond to resonant field lines with successive numbers of nodes. In the toroidal case, the entire mode consists of one singular layer, its location being accurately given by a resonant field line. As a result of this singular structure, the damping rate of toroidal modes decreases more slowly than that of poloidal modes when diffusivities are decreased. In both cases the least-damped modes are the closest to the magnetic poles. Fig. 1 shows a poloidal and a toroidal mode and Fig. 2 shows frequencies at which different field lines resonate.

An interesting aspect of this problem is the quantization of the eigenvalue spectrum. Both the axisymmetric poloidal and toroidal spectra display a double quantization. The first quantization corresponds to the number of nodes n along field lines. This is represented by the gaps between the horizontal branches in Fig. 3. The second quantization applies to the horizontal structure of the mode and differs somewhat between the poloidal case (where the structure is regular) and the toroidal case (where the mode is singular). It leads to the different modes along a given branch. In the non-axisymmetric case, the poloidal and toroidal spectra are mixed together, leading to branches that are doubled up. However, when diffusivities are small enough, these branches start taking on more distinctive poloidal or toroidal behaviours.

Due to the structure of the toroidal part of the equations, it is possible to come up with a first order analytic solution to them, which has following basic form:

$$
\begin{array}{lcl}
\lambda_{n,q} & = & \lambda_n^0 + \varepsilon^{1/2}\lambda_{n,q}^1 \text{ where } \lambda_{n,q}^1 = (q+1)\tilde{\lambda}_n^1, \\
b_\varphi & = & b_n^0(r) f_q(\nu), \\
v_\varphi & = & v_n^0(r) f_q(\nu),
\end{array}
\tag{3.1}
$$

where q represents the second quantum number, r is the radial coordinate, ν is a dipolar coordinate that remains constant along field lines, and ε a small parameter

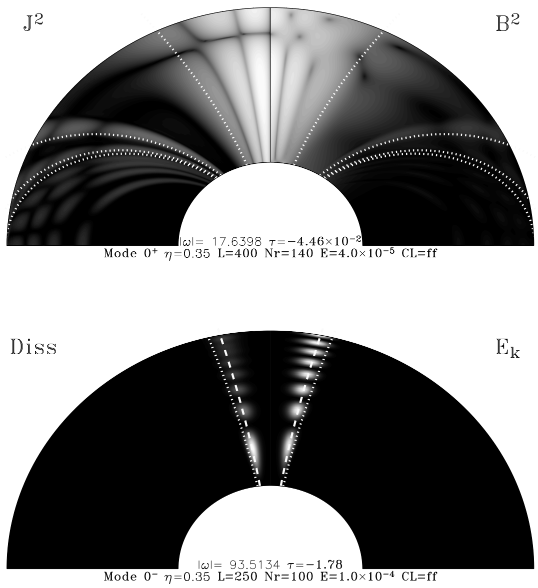

Figure 1. The upper figure corresponds to a poloidal mode and the lower one to a toroidal mode. The dotted lines correspond to resonant field lines calculated with a WKB approximation, whereas the dashed line in the lower figure is a more precise calculation adapted only to toroidal modes. Internal shear layers near the dotted lines are slightly visible in the poloidal mode. The lower figure uses a linear intensity scale instead of logarithmic one in order to give the mode a thinner appearance.

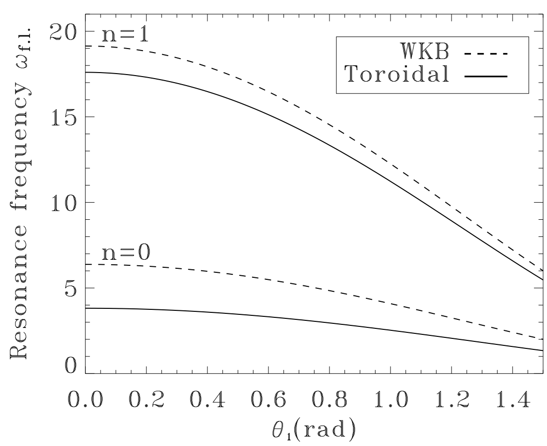

Figure 2. Resonance frequencies of different field lines for $n = 0$ and $n = 1$ node. The angle θ_1 gives the colatitude of the field line as it crosses the outer boundary. For a given number of nodes, the oscillation rate decreases when moving away from the magnetic poles.

proportional to E and E_{m}. The expressions of λ_n^0, $\lambda_{n,q}^1$, b_n^0, v_n^0 and f_q are given in Reese et al. (2004). These formulas give accurate frequencies ω and a good idea of the modes' structure (see Fig. 4). However the damping rate τ predicted is not very accurate due to a term that is $\mathcal{O}(\varepsilon)$ but proportional to ω^2 (see Fig. 3).

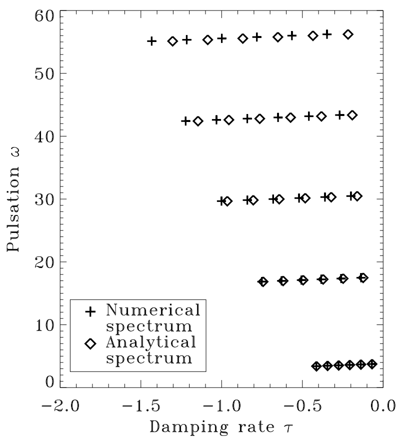

Figure 3. A comparison between a numerical eigenspectrum and an analytical one based on Eq. (3.1). The damping rate differs by a term proportional to $\varepsilon\omega^2$. The error on the frequencies ranges from 1×10^{-4} to 1.4×10^{-3}.

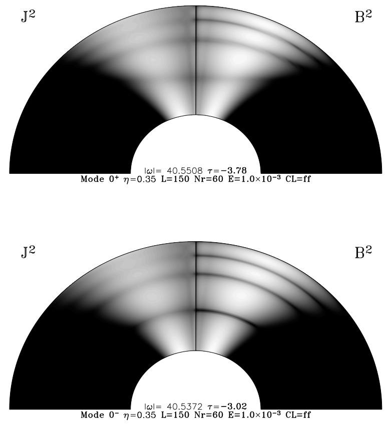

Figure 4. Comparison between a numerical mode (upper figure) and an analytical one (lower figure). The left quadrant of both figures is the magnetic dissipation and the right quadrant the magnetic energy.

4 Conclusion

In this work, we have explored some of the basic properties and characteristics of magnetic eigenoscillations of a highly simplified "star". These studies show the existence of two types of oscillatory modes, a double quantization of the eigenvalue spectrum, and an approximate analytical solution in the toroidal case. It is also interesting to note the importance of the magnetic poles for the least damped modes, which is coherent with current observations of roAp stars. In subsequent work, more realistic models of stars will be envisaged, and compressibility will be taken into account in order to analyse magneto-acoustic waves.

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